Journal of Computer Applications ISSN: 0974 – 1925,
Volume-5, Issue EICA2012-1, February 10, 2012

Skeletal Graph Based Topological Feature Extraction of an Object

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Abstract - The propose a novel algorithm that compute a skeletal graph and thus capture the topology of an object Topology is an important attribute of an object which describes how different parts of an object surface are connected to each other. The method is based on capturing the topology of a modified reeb graph by tracking the critical points of a distance function. The algorithm for constructing the distance function based skeletal graph follows directly from the Morse lemma, which states that a change in the topology of a level set of a Morse function occurs only at its critical level. Distance function is used for constructing skeletal graphs. This approach employs Morse theory in the study of translation, rotation, and scale invariant skeletal graph.

Keywords : *Morse theory, Distance function, Skeletonization.*

I. INTRODUCTION

One approach to finding the skeleton of a binary 2 Dimensional (2D) image is to apply a distance transformation to the image and locate the “crests” of the resulting distance map. The distance transform finds the distance between all non-boundary foreground points in an image and their nearest boundary points. The distance between points may be defined using a 4-connected neighborhood or an 8-connected neighborhood. When using an 8-connected neighborhood, the distance between diagonal points may be defined to be the same as the distance between horizontal and vertical points, or it may be defined to be larger. The result of applying a distance transformation is a distance map. Once a distance map is acquired, it is useful to draw a representation of the result where the gray level of a pixel represents the distance recorded for a particular point. The distance transformation can be executed in linear time with respect to the size of the foreground region. The algorithm therefore has quadratic complexity with respect to a single dimension of the image. Morse theory provides the basic framework for topological analysis of smooth manifolds. Morse theory relates the topology of a smooth manifold with the number of critical points of a Morse function defined on this manifold. A k-dimensional manifold M may be locally parameterized as

$$\varphi: \Omega \rightarrow M \qquad (1)$$

where an open connected set $\Omega \subset R^k$ represents the parameter space. Let f: $M \rightarrow R$ be a real-valued function defined on M. By definition, the function f is smooth if the composition $f \circ \varphi: \Omega \rightarrow R$ is smooth for each local parameterization of M. A point $x = \varphi(u) \in M$, where

$u \in \Omega$ is called a critical point of f if the gradient of $f \circ \varphi$ vanishes at u.

The Morse Lemma states that there exists a parameterization of a neighborhood of a non-degenerate critical point of f in which $f \circ \varphi$ attains a quadratic form. For instance, the function $f(x) = x^2$ has a non-degenerate critical point at $x = 0$, which is in accordance with the local quadraticity of the function.

If f is a smooth function on a two-dimensional manifold M, three possible types of non-degenerate critical points exist, namely the local minimum (index 0), the saddle point (index 1), and the local maximum (index 2).

Definition (Morse function):

A smooth function $f: M \to R$ on a smooth manifold M is called a Morse function if all of its critical points are non-degenerate.

A Morse function satisfies the following basic properties:

- Critical points of a Morse function are isolated.
- The number of critical points of a Morse function is stable, that is, a small perturbation of the function neither creates nor destroys critical points.
- The number of critical points of a Morse function on a compact manifold is finite.

The level set $L_t = f^{-1}(t) \subset M$ of the Morse function $f: M \to R$ is called critical, if it contains a critical point of f. According to the Morse Deformation Lemma, if any two levels L_{c1} and L_{c2} have different topological types, there is a number $c \in (c_1, c_2)$ such that L_c is a critical level. In other words, a change of topology occurs only at a critical point.

Example: 1 (The Height Function on a Sphere):

The height function defined on a unit sphere $M = S^2$ is a real-valued function $h: M \to R$ such that $h(x, y, z) = z$, $\forall (x, y, z) \in M$. This function has two critical points, minimum at the South Pole and maximum at the North Pole. It is straightforward to show that both are non-degenerate, indicating that h is a Morse function.

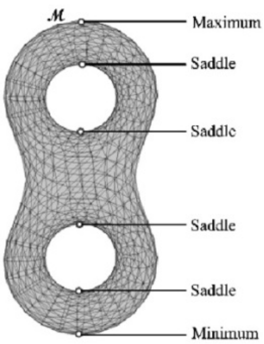

Figure 1: Critical Points of a Height Function Defined on a Manifold M.

II. RELATED WORK

Several approaches to capturing the topology of a surface are made available.

- **Point Correspondences:** Many algorithmic approaches are based on establishing point correspondences between two shapes. In [9], Sclaro and Pent land proposed a point correspondence method: the correspondences were established in a generalized feature space which is determined by eigenmodes of a finite element representation of a shape. The resulting correspondence was shown to be invariant under ne transformations and insensitive to noise. This method, however, is highly global and operates on a shape contour as a whole without taking local features into account.

- **Medial Axis:** Another class of computational geometry-based methods is that of medial axis representation of shapes [10, 11]. These models owe much to their simplicity which is also their limitation of capturing variability across various shapes. Specifically, medial axis methods may lead to a non-unique representation of a shape. Zhu and Yuille presented a method and referred to it as FORMS, which is based on a variant of medial axis [12]. Their model involves two primitives, which when deformed yield what are called mid-grained shapes which in turn capture parts of an object. These mid-grained shapes are in the end assembled to represent complete objects by using a custom grammar. The dependence of such a model on primitives and its complexity due to the burden of grammar rules reduces its flexibility.

- **Shape Axis:** Liu et al. proposed a method for shape recognition via matching shape axis trees which are derived from the shape axis [13]. The shape axis is similar to the medial axis and is defined as the locus of midpoints of corresponding boundary points on two given shapes. Shape axis trees are then modified to achieve the best match reflected by an associated cost which is based on the approach in [14]. Although this method addresses articulations and occlusions, it has limitations similar to those in [15].

- **Shock Graphs:** Shock graphs are a variant of the medial axis, as they capture its evolution in time. Specifically, the shock graph is a locus of singularities (shocks) formed by a propagating wave (grass-fire) from the boundaries [12]. A shock graph may be viewed as a medial axis endowed with additional information. Hence, it may result in a unique representation over a wider class of shapes and is, therefore, generally regarded as a better shape descriptor with numerous variants.

Shinagawa et al. [1] proposed a Reeb graph representation by utilizing the height function. In addition to the lack of rich geometric information, a fundamental limitation of this approach was its dependence on the surface orientation. Nonetheless, its framework, based on the elegant and mathematically sound Morse theory, opened up promising new avenues for geometrically richer object representations. Lazarus et al. [2], for instance, proposed level set diagrams, which were driven by geodesic distance from a manually chosen source point. Hilaga et al. [3] extended level set diagrams in the form of multiresolution Reeb graphs, which eliminated the need of the manual source point. In addition, their matching algorithm was driven by richer geometric information. Geometric point features were proposed by Tung et al. [4] to augment MRGs, and a more comprehensive representation building on [3] was recently proposed by Aouada et al. [5].

III. TOPOLOGICAL MODEL

Consider the distance function d: p → ‖ ‖p ‖ ‖ in R^3. Given a generic surface $M \subset R^3$, the restriction of the distance function on M,

$$d: M \rightarrow R_+ \quad \mathbf{(2)}$$

is a Morse function, i.e., all critical points of d on M are non-degenerate. One can thus use the distance function for constructing a skeletal graph of the surface M.

To analyze and encode a compact surface using the Morse function, the start at the origin and gradually increase the value of the distance function in K steps to a sufficiently large number which are denote b. The integer is called the resolution of the skeletal graph. Making K larger increases the precision of captured structural changes in the level sets of the distance function. Recall that such changes occur only at critical level sets.

Since the level sets of d are concentric spheres, the find intersections of the manifold with spheres of radii R for all $R \in [0, b]$ and assign a node to each connected component in an intersection. The skeletal graph may be described as the quotient space $M/\sim$, where the equivalence relation $\sim$ is defined below.

Definition (Equivalence):

To say that the points p and q on the surface are equivalent and write $p \sim q$ if and only if p and q belong to the same connected component of the level set of the function d.

Recall the definition of the quotient space: $M/\sim := \{[p] \mid p \in M\}$, where the equivalence class [p] of the point $p \in M$ is the set of all points $q \in M$ such that $q \sim p$.

Note that the function d given by Eq. (1) is not invariant with respect to translation and scaling. In order to have this invariance, we put the origin at the centroid μ of the surface of interest and set

$$d_\mu (p) := p - \mu \quad (3)$$

We can introduce scale invariance through the following transformation:

$$\tilde{d}_\mu (p) = \frac{d^{\mu (p)} - d_{min}}{d_{max} - d_{min}} \quad (4)$$

Proposition 1 (Invariance):

The distance function given is rotation, translation and scale invariant. The above proposition demonstrates the invariance of the distance function to rigid body transformation under the condition that the centroid of the manifold must be translated to the origin.

A. Skeletonization Algorithm

Skeletonization is a transformation technique applied to an object whereby the object is reduced to a single dimensional representation, called the skeleton of the object. The transformation applies to real-space objects as well as objects in discrete space. A naive implementation of skeletonization is very inefficient. For each point p within the foreground of an image, the nearest neighbor(s) of p must be located within the boundary points of the image. If there are m points along the boundary, finding the nearest neighbor of p can take O (m) time if the boundary points are already known. If they are not, the process can take O (wh) time, where w and h are the width and height of the image, respectively. Finding all nearest neighbors can thus take O (nwh) time, where n is the number of foreground points. Since n = O (wh), the worst case runtime of the naive approach is O (w_2h_2), which is quadratic with respect to a single dimension of the image.

The skeleton is useful because it provides a simple and compact representation of a shape that preserves many of the topological and size characteristics of the original shape. Thus, for instance, I can get a rough idea of the length of a shape by considering just the end points of the skeleton and finding the maximally separated pair of end points on the skeleton. Similarly, I can distinguish many qualitatively different shapes from one another on the basis of how many `triple points' there are, *i.e.* points where at least three branches of the skeleton meet.

The steps involved in Skeletonization algorithm are as follows:

1. Find the centroid of the surface M, place the origin at the centroid
2. Find dmax, the maximum distance from the centroid to M
3. Given K, define: r_k: = k dmax/K, k = 1. . . K
4. Generate the spheres S1 and S2 with radii R = r1 and R = r2, respectively
5. Find ˜Mp =M n ([S1] n [S2]), identify the interior and exterior of a closed surface M p is, therefore, the part of M that lies between S1 and S2
6. Assign a node N_{Mp} to each connected component Mp of ˜Mp at the centroid of Mp
7. For k = 3 to K Generate the "current" sphere S_k with radius R = r_k
8. Find ˜Mc =M n ([S_{k-1}] n [S_k]). Hence, ˜Mc is the portion of M that lies in between S_{k-1} and S_k
9. Find the connected components Mc of ˜Mc

    For each Mc belong to ˜Mc do
    - Assign a node NMc at the centroid of Mc
    - Find the connected region Mp belong to ˜Mp such that Mc contained in Mp is a single connected region.
    - Add an edge between N_{Mc} and N_{Mp}

    end for

˜Mp = ˜Mc
end for.

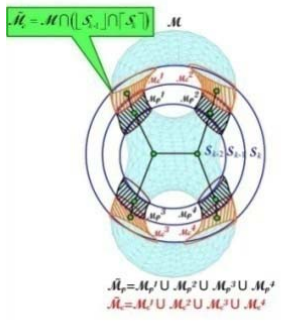

Figure 2 : Skeletonization of a surface M.

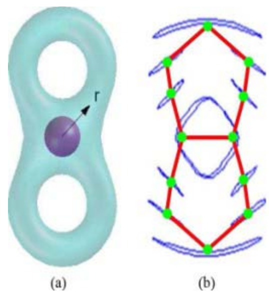

Figure 3: Skeletal graph of a double torus: (a) an evolving sphere; (b) level curves and node assignment.

IV. EXPERIMENTAL RESULT

Skeletonization is a morphological operation that is used to remove selected foreground pixels from binary images. Skeletonization is normally only applied to binary images, and produces another binary image as output. The term ‘skeleton’ has been used in general to denote a representation of a pattern by a collection of thin arcs and curves. Other nomenclatures have been used in different context. Thus, skeletonization is defined as process of reducing the width of pattern to just a single pixel. Skeletal graph of the objects are given as follows in figure

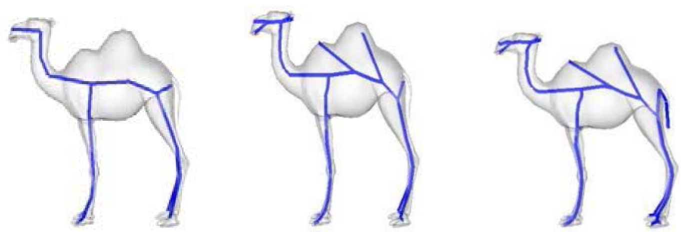

Figure 4 : Skeletal graphs for a camel: (a) K = 8; (b) K = 16; (c) K = 32.

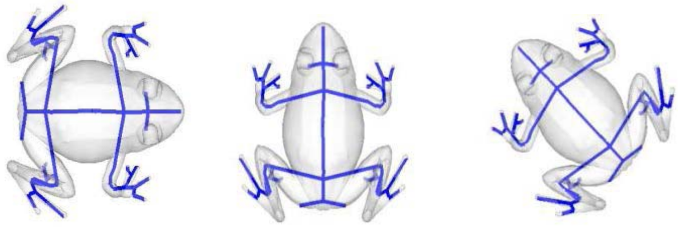

Figure 5: Rotational invariance of a skeletal graph: (a) No rotation; (b) Rotation by ; (c) Rotation by

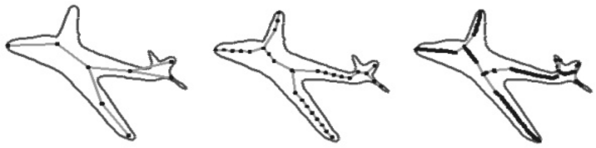

Figure 6: Skeletonization of an airplane: (a) $K = 4$; (b) $K = 16$; (c) $K = 64$.

V. CONCLUSION

Skeletonization is a useful technique. It can be applied to object recognition and image analysis problems. Several algorithms exist for computing the skeleton of an image in discrete 2D space. The technique of distance transformations is reportedly very efficient. Voronoi diagrams can be used to find a medial axis of a region, but complicated procedures must be added to the implementation to ensure only the salient features of a region are represented in the skeleton. The Object Recognition through skeletonization greatly reduces the amount of time and data for processing. In this paper, proposed a skeletal graph representation of an object that is rotation, scale and translation invariant. This skeletal graph is used for object recognition and compression.

REFERENCES

1. Aouada D., Dreisigmeyer D.W., and Krim H., "Geometric modeling of rigid and nonrigid 3D shapes using the global geodesic function," presented at the IEEE CVPR NORDIA Workshop, 2008.
2. Baloch S., Krim H., Zenkov D., and Kogan I., "3D object representation with topo-geometric shape models," presented at the EUSIPCO, 2005.
3. Baloch S.H., Krim H., Kogan I., and Zenkov D., *Topo-Geometric Shape Model for 3D Object Representation,* under review.
4. Basri R, Costa L, Geiger D, Jacobs D (1998) Determining the similarity of deformable shapes, *Vision Research*, Vol. 38, pp. 2365–2385.
5. Blum H (1967) A transformation for extracting new descriptors of shape, In: Whaten-Dunn (ed) *Models for the Perception of Speech and Visual Forms*, pp. 362–380, MIT Press, Cambridge, MA.
6. Hilaga M., Shinagawa Y., Kohmura T., and Kunii T.L., "Topology matching for fully automatic similarity estimation of 3D shapes," *Proc. SIGGRAPH*, pp. 203–212, 2001.
7. Hiransakolwong N, Vu K, Hua KA, Lang SD (2004) Shape Recognition Based on the Medial Axis Approach, *Proceedings of ICME'04*, Taipei, Taiwan, June 27–30.

8. Kimia B B, Tannenbaum AR, Zucker SW (1995) Shapes, shocks and deformations, I: the components of shape and the reaction-diffusion space, *International Journal of Computer Vision*, Vol. 15, No. 3, pp. 189–224.
9. Lazarus F., and Verroust A., "Level set diagrams of polyhedral objects," in *Proc. 5th ACM Symp. Solid Modeling and Applications*, 1999, pp. 130–140.
10. Leventon M E, Grimson WEL, Faugeras O (2000) Statistical shape influence in geodesic active contours, *Proceedings CVPR'00*, Vol. 1, pp. 316–323.
11. Liu TL, Geiger D, Kohn RV (1998) Representation and Self-Similarity of Shapes, *International Conference on Computer Vision*, Bombay, India.
12. Sajjad Baloch, Member, IEEE, and Hamid Krim, Fellow (May 2010), "Object Recognition through Topo-Geometric Shape Models Using Error-Tolerant Sub graph Isomorphism" in IEEE TRANSACTIONS ON IMAGE PROCESSING, vol. 19, no. 5.
13. Shinagawa Y., and Kunii T.L., "Constructing a Reeb graph automatically from cross sections," *IEEE Computer. Graph. Appl.*, vol. 11, no. 6, pp. 44–51, 1991.
14. Tung T. and Schmitt F., "Augmented Reeb graphs for content-based retrieval of 3D mesh models," *Shape Model. Int.*, pp. 157–166, 2004.
15. Zhu SC, Yuille AL (1996) FORMS: A flexible object recognition and modeling systems, *International Journal of Computer Vision*, Vol. 20, No. 3, pp. 187–212.