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Fuzzy Fault Tree Analysis – A Case Study

Mamatha S Upadhya[a,*]

***Abstract* - A useful tool in performing a system safety analysis is fault tree analysis. In this paper, the fault tree analysis (FTA) using fuzzy probability is explained. This enables the analyser to incorporate the expert opinion by means of fuzzy membership functions into the analysis. A case study in a medical treatment is considered. The analysis for occurrence of the top event i.e., 'Patient contracts bacterial infection during therapy' is carried out. Firstly the fault tree is drawn and then the fault tree is analysed using fuzzy membership functions [1].**

Index Terms – Fuzzy Logic, Fuzzy Membership Function, Fault Tree Analysis.

I. INTRODUCTION

The product design involves reliability analysis as one of its major tasks. The reliability analysis comprises of Fault Tree Analysis (FTA) as one of the important activities among the reliability tasks of an item design. A fault tree analysis is a graphical design technique for analyzing the top event, which causes the system failure. It is a top-down, deductive analysis structured in terms of events leading to the occurrence of the top event. FTA is a useful tool in performing a system safety analysis.

II. FAULT TREE ANALYSIS (FTA)

Fault tree analysis is a systems engineering technique, which provides an organized, illustrative approach to the identification of high-risk areas. This technique was developed in 1961 at Bell Telephone Laboratories. A fault tree diagram follows a top down structure and represents a graphical model of the pathways within a system that can lead to a foreseeable, undesirable loss event or a failure. The pathways interconnect contributory events and conditions using standard logic symbols (AND, OR, etc). If the occurrence of either input event causes the output event to occur, then these input events would be connected using OR gate. On the other hand, if both input events must occur in order for the output event to occur, then they are connected using AND gate. Construction of a fault tree usually begins with the definition of the top undesired event (the system failure). The causes are then indicated and connected to the top event by logic gates.

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Mamatha S Upadhya,
Department of Mathematics,
Garden City College, Bangalore 560049, India
E-mail: adithi9@hotmail.com

The procedure is repeated for each of the causes and the causes of the causes, etc., until all the events have been considered. Furthermore, given the probability of occurrence of the basic events, the probability of occurrence of the top undesired event could be calculated. This can be accomplished using several approaches such as cut-set approach and Bayesian approach. Hence the FTA consists of two parts: construction and evaluation of fault tree [2, 3].

III. FAULT TREE ANALYSIS (FTA) USING FUZZY PROBABILITIES

Though FTA was developed long back, still it cannot be performed functionally without facing imprecise failure input data and improper modeling problems. Towards this end, fuzzy sets can help to overcome this situation. Experts utilize fuzzy sets to subjectively describe the uncertainties of each given failure event, and then perform mathematical operation to evaluate system reliability. The failure events are modelled to be trapezoidal fuzzy sets [1]. These fuzzy sets are considered as the possibility of occurrence of the failure events. Therefore the problem is to calculate the possibility of failure of the top event as a fuzzy set, given the possibilities of failure of the basic events.

The possibility of failure of an event X_i represented by the trapezoidal fuzzy set is:

$$\tilde{P}_{X_i} \equiv (q_i^l, p_i^l, p_i^r, q_i^r) \qquad (1)$$

This is defined by the membership function:

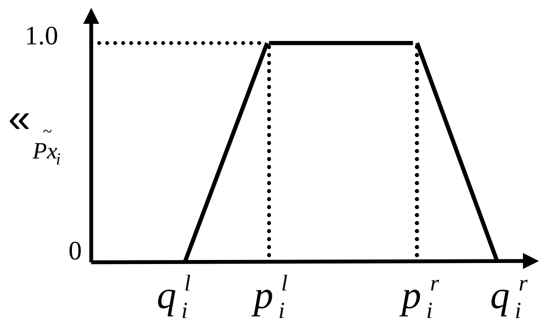

Figure1. Trapezoidal Fuzzy Membership Function for $\tilde{P}_{X_i}$

$$« _{\tilde{P}x_i}(p) = \begin{cases} 0; & 0 \le p \le q_i^l \\ 1-{}^{-}{}_i^l; & q_i^l \le p \le p_i^l \\ 1; & p_i^l \le p \le p_i^r \\ 1+{}^{-}{}_i^r; & p_i^r \le p \le q_i^r \\ 0; & q_i^r \le p \le 1 \end{cases}$$

with $\Delta p_i^{\sim} \equiv p_i^{\sim} - q_i^{\sim}, \quad \sim = l,\ r$ and $-_{i}^{\sim} \equiv \dfrac{p_i^{\sim} - p}{\Delta p_i^{\sim}}$

As the multiplication of two fuzzy sets is complicated, the following approximate product ∘ is used in this article for multiplication of two fuzzy sets [1].

$$\tilde{P}_{X_i} \circ \tilde{P}_{X_j} \equiv (q_i^l q_j^l,\ p_i^l p_j^l,\ p_i^r p_j^r,\ q_i^r q_j^r) \approx \tilde{P}_{X_i}\ \tilde{P}_{X_j} \qquad (2)$$

$$1 - \tilde{P}_{X_i} \equiv (1 - q_i^r,\ 1 - p_i^r,\ 1 - p_i^l,\ 1 - q_i^l) \qquad (3)$$

The fuzzy fault tree analysis can be performed with the following steps:

a. Identify the top event which causes the system failure
b. Analyse all possible events and causes for the system failure in terms its failure of subsystems and components.
c. Construct the fault tree using logic gates such as AND and OR gates.
d. Using the minimal cut-set approach, obtain the probability model for the occurrence of the top event in terms of the probability of occurrence of the basic events.
e. Obtain the fuzzy probabilities for the occurrence of the basic events in terms of equation (1).
f. Using the probability model obtained in (iv) and equations (2) and (3), calculate the fuzzy probability for the occurrence of the top event.

For simplicity, define

$$\tilde{P}_T(\tilde{P}_{X_1}, ..., \tilde{P}_{X_i}, ..., \tilde{P}_{X_n}) \equiv \tilde{P}_T$$ and

$$\tilde{P}_T(\tilde{P}_{X_1}, ..., \tilde{P}_{X_{i-1}}, 0, \tilde{P}_{X_{i+1}} ..., \tilde{P}_{X_n}) \equiv \tilde{P}_{T_i}$$

The index V, which measures the difference between $\tilde{P}_T$ and $\tilde{P}_{T_i}$ is defined below [1] and this indicates the extent of improvement in eliminating the occurrence of the event X_i.

$$V(\tilde{P}_T, \tilde{P}_{T_i}) \equiv (q_T^l - q_{T_i}^l) + (p_T^l - p_{T_i}^l) + (p_T^r - p_{T_i}^r) + (q_T^r - q_{T_i}^r) > 0 \qquad (4)$$

For example, if $V(\tilde{P}_T, \tilde{P}_{T_i}) \geq V(\tilde{P}_T, \tilde{P}_{T_j})$, then eliminating X_i is more effective than removing X_j. Using this index, we can rank the basic events according to the degree of effectiveness in improving the system.

IV. CASE STUDY ON PATIENT CONTRACTING BACTERIAL INFECTION DURING THERAPY

Consider a typical medical treatment example wherein a patient can contract bacterial infection during therapy. The case is analysed by means of a Fault Tree. Taking the top event as 'Patient contracts bacterial infection during therapy', the fault tree is drawn with all possible events leading to the top event. The identified fault events are logically connected by means of logic gates such as 'AND and 'OR' gates. The fault tree thus derived is given in Figure 2.

The probability of occurrence of the top event i.e., 'Patient contracts bacterial infection during therapy' can be calculated using the definition of the logic gates given the probability of occurrence of the basic events. The minimal cut sets of the fault tree in terms of the basic events are X_3 and X_1 X_2, where

X_1 : Poor hygiene
X_2 : Lack of training
X_3 : Human error

If P_T is the probability of occurrence of the top event and P_{X_i}, i = 1, 2, 3 is the probability of occurrence of the X_i^{th} basic event, then

$$P_T = P_{X_3} + P_{X_1} P_{X_2} - P_{X_1} P_{X_2} P_{X_3} \qquad (5)$$

Now, (5) can be written as

$$P_T = 1 - (1 - P_{X_3})(1 - P_{X_1} P_{X_2}) \qquad (6)$$

The fuzzy sets considered for the basic events of the minimal cut sets are given below. This has been derived based on the expert opinion.

$$\tilde{P}_{X_1} \equiv (0.01, 0.004, 0.008, 0.01)$$

$$\tilde{P}_{X_2} \equiv (0.01, 0.015, 0.035, 0.05)$$

$$\tilde{P}_{X_3} \equiv (0.01, 0.013, 0.018, 0.02) \qquad (7)$$

Using the approximate product ∘ the fuzzy set corresponding to the top event can be written as:

$$\tilde{P}_T = 1 - [(1 - \tilde{P}_{X_3}) \circ (1 - \tilde{P}_{X_1} \circ \tilde{P}_{X_2})] \qquad (8)$$

Using (2), (3), (6), (7) and (8) the fuzzy set corresponding to the top event is calculated to be: $\tilde{P}_T \equiv (0.0101, 0.0131, 0.0183, 0.0205)$ Hence the possibility of occurrence of top event X_T is obtained as the trapezoidal fuzzy set: $\tilde{P}_T \equiv (0.0101, 0.0131, 0.0183, 0.0205)$ and its membership function is given by

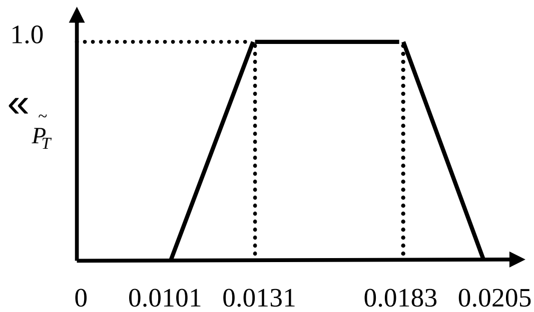

Figure 3. Trapezoidal Fuzzy Membership Function for $\tilde{P}_T$

With prevention of the occurrence of fundamental events X_i, i = 1, 2, 3 that contribute to the minimal cut sets, the corresponding fuzzy probability of top event,

$\tilde{P}_{T_i}$, $i = 1, 2, 3$ and $V(\tilde{P}_T, \tilde{P}_{T_i})$, $i = 1, 2, 3$ are obtained and listed in Table 1. The values of $V(\tilde{P}_T, \tilde{P}_{T_i})$ indicate that preventing the occurrence of X_3 i.e., the human error is most effective; this yields

$$\tilde{P}_{T_3} = (0.0001, 0.00006, 0.00028, 0.0005).$$

V. Conclusions

A useful tool in performing a system safety analysis is fault tree analysis. The focus is usually on a significant failure or a catastrophic event, which is referred to as the top event. The analysis consists of identifying the various combinations of events that will cause the top event to occur. This is then followed by a quantitative analysis to estimate the probability of occurrence of the top event. The quantitative analysis requires probability of occurrence of the basic fault events. Many a times it becomes difficult to quantify such probabilities. In such cases experts utilize fuzzy sets to subjectively describe the uncertainties of each given basic failure event. In this paper, trapezoidal fuzzy sets were used to subjectively assess the occurrence of the basic events. Using these, the fuzzy probability in terms of the trapezoidal fuzzy set for the top event i.e., Patient contracts bacterial infection during therapy is obtained. The results obtained give an idea of the fuzzy distribution of the Patient contracting bacterial infection during therapy for the considered fuzzy probabilities of the basic failure events. Further, using the index V it is found that preventing the human error is most effective in terms of the fuzzy probability of failure of the top event.

Table 1: $\tilde{P}_{T_i}$ and $V(\tilde{P}_T, \tilde{P}_{T_i})$

$\tilde{P}$	$\tilde{P}_{T_i} \equiv (q_i^l, p_i^l, p_i^r, q_i^r)$	$V(\tilde{P}_T, \tilde{P}_{T_i})$
$\tilde{P}_{T_3}$	(0.0001, 0.00006, 0.00028, 0.0005)	0.0611
$\tilde{P}_{T_2}$	(0.01, 0.013, 0.018, 0.02)	0.001
$\tilde{P}_{T_1}$	(0.01, 0.013, 0.018, 0.02)	0.001

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Biography

Mamatha S Upadhya holds M.Sc. (Mathematics) from Kuvempu University, Karnataka and M.Phil. (Mathematics) from S.V University, Tirupathi. She has nearly 10 years of teaching experience for both undergraduate and postgraduate courses. Currently she is working as Assistant Professor at the Garden City College, Bangalore. Her areas of interest include Graph Theory, Fuzzy logic, Differential Calculus etc.

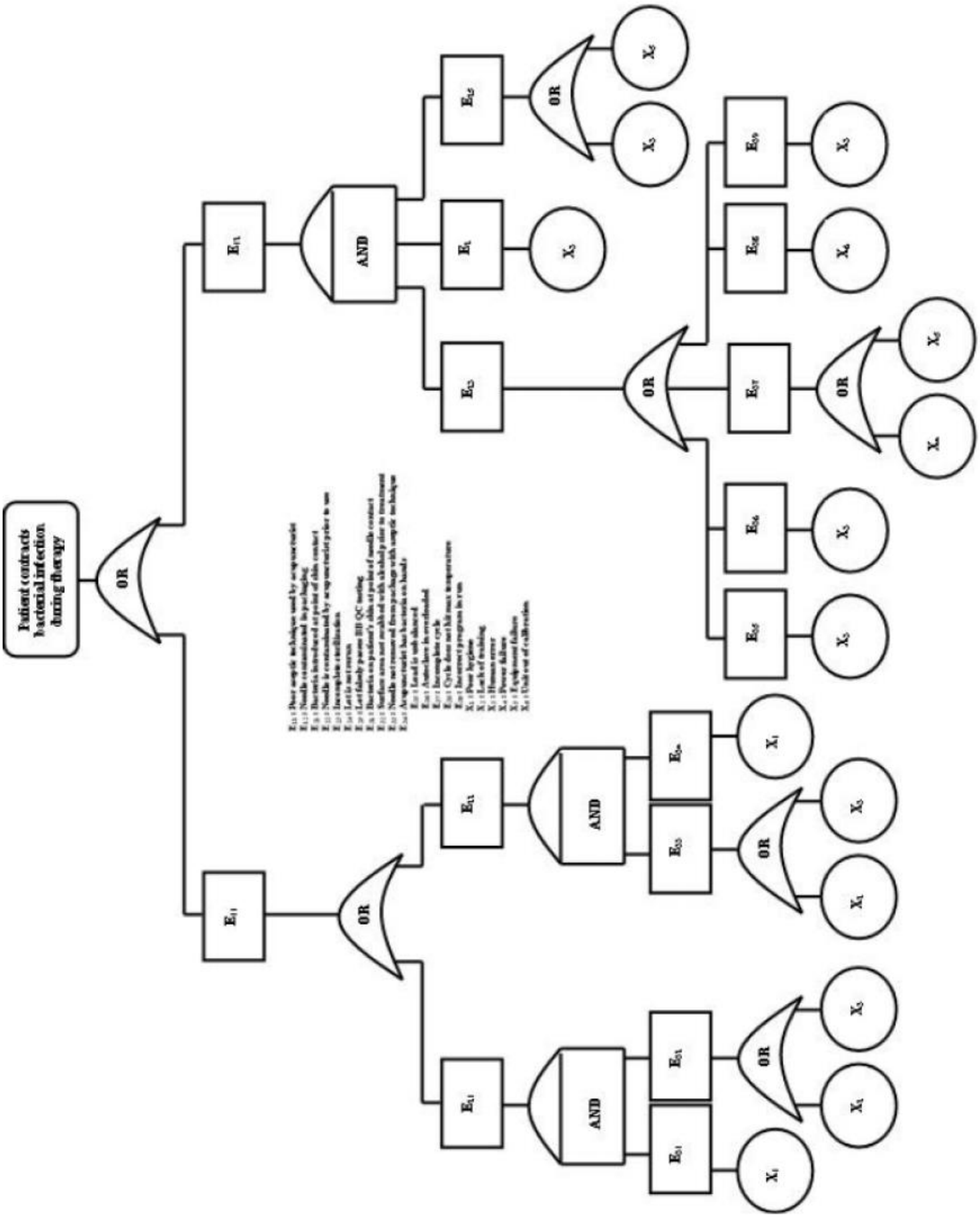

Figure 2. Fault Tree Diagram for 'Patient Contracts Bacterial Infection During Therapy'